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TRIVANDRUM

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme Under CBCSS

Mathematics

Core Course X

MM 1642 – COMPLEX ANALYSIS II

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries **1** mark.

1. When does a complex series diverge?
2. Define Uniform Convergence of a series of functions.
3. Explain the term: Cauchy sequence.
4. Define a zero of order m for a function f.
5. What is the formula for

   $\mathrm{Res}(f;z_0)$, if $f(z)=\frac{p(z)}{Q(z)}$, where $P(z)$ and $Q(z)$ are analytic at z_0

   and Q has a simple pole at z_0, while $P(z_0)\neq 0$.
6. Define Improper integral over $(-\infty,0)$ of a continuous function $f(x)$.

7. Define p.v. $\int_{-\infty}^{\infty} f(x)dx$.

8. What do you mean by global property of a mapping?

9. Are analytic functions that are non constant in domains being open mappings?

10. Define the mapping : Rotation.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. Prove that $1+c+c^2+c^3+\cdots=\frac{1}{1-c}$ for $|c|<1$.

12. Find the Maclaurin series for cos *z*.

13. How can we obtain the Taylor series of *fg*, if *f* and *g* are two analytic functions?

14. State a necessary and sufficient condition for the convergence of a sequence of complex numbers.

15. Explain the term: Laurent Series

16. State the necessary and sufficient condition for an analytic function to have a pole of order *m* at z_0.

17. State the Picard's Theorem.

18. Define of a residue of a function with an example.

19. Find the residue at $z=0$ of $f(z)=ze^{\frac{3}{z}}$ using Laurent series.

20. Find the residue at $z=-3i$ of $f(z)=\frac{z+1}{z^2+9}$.

21. Evaluate *p.v.* $\int_{-\infty}^{\infty} x^3dx$.

22. Give the statement of Jordan's Lemma.

23. Is $f(z)=z^2$ being locally one to one on a neighbourhood of 0? Justify your answer.

24. Write the statement of Riemann Mapping Theorem.

25. Define magnification and show that it rescales distance.

26. State the symmetry principle of Mobius transformations.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. **Each** question carries **4** marks.

27. State the Ratio test and show that $\sum_{j=1}^{\infty} \frac{4^j}{j!}$ converges.

28. If $f(z)$ is analytic at z_0, how can we find the Taylor series of $f'(z)$. Using this result, find the Taylor series of cos z from that of $z = \sum_{j=1}^{\infty} \frac{z^{2j-1}}{(2j-1)!}$.

29. Prove that the uniform limit of a sequence of analytic functions defined on a simply connected domain is also analytic.

30. Classify the zeros and singularities of $\frac{\tan z}{z}$.

31. What do you mean by extended complex plane? Classify the behaviour at ∞ of $f(z) = \frac{iz+1}{z-1}$.

32. Find the residues at each singularity of $f(z)$ = *cosec z*.

33. State and prove Cauchy Residue Theorem.

34. Prove that if $|f(t)| \le M(t)$ on $a \le t \le b$, then $\left| \int_a^b f(t)dt \right| \le \int_a^b M(t)dt$ where $f(t)$ and $M(t)$ are continuous function defined on [*a, b*], with *f* complex and *M* real valued.

35. Find the integral of $\frac{\sin x}{x}$ over $(0,\infty)$.

36. Prove that if $f(z)$ is analytic at z_0, then there is an open disk D centred at z_0 such that f is one to one on D.

37. Define linear transformation and find the linear transformation that maps the circle $|z-1|=1$ onto the circle $\left|w-\frac{3i}{2}\right|=2$.

38. Show that the composition of two Mobius transforms is another Mobius Transform.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. **Each** question carries **15** marks.

39. Define Taylor Series of $f(z)$ around z_0. Prove that if $f(z)$ is analytic in the $|z-z_0|<R$ there exists a Taylor Series which converges to $f(z)$ for all z in this disk.

40. Find The Laurent series expansion of $f(z)=\frac{1}{(z-1)(z-2)}$ in

(a) The region $|Z|<1$

(b) The region $1<|Z|<2$

(c) The region $|Z|>2$.

41. Define different singularities of a complex function with examples. Verify the examples with definition.

42. (a) Prove that, if $f(z)$ has a pole of order m at z_0, then

$$\operatorname{Res}(f:z_0)=\lim_{z\to z_0}\frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\left[(z-z_0)^m f(z)\right]$$

(b) Using the result find the residue at $z=0$ of $f(z)=\frac{\cos z}{z^2(z-\pi)^3}$.

43. Evaluate $\int_0^x \frac{1}{2-\cos\theta}d\theta$.

44. (a) Prove that if $f(z)$ is analytic at z_0 and $f'(z_0)\neq 0$ then $f(z)$ is conformal at z_0.

(b) Find all Mobius transformations that map unit disks into unit disks.

(2 × 15 = 30 Marks)
